

Heavy Quark Theory[★]

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Abstract

Recent progress in the theory of hadrons containing a single heavy quark is reviewed. Particular attention is paid to those aspects that bear on the determination of the magnitudes of the Cabibbo–Kobayashi–Maskawa matrix elements V_{cb} and V_{ub} .

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1. Introduction

Over the past year there have been several important developments in the theory of hadrons containing a single heavy quark. At the same time there have been significant improvements from experiment in our understanding of the properties of hadrons containing a charm or bottom quark.

The minimal standard model has six quarks that couple to the charged W -bosons through the term

$$\mathcal{L}_{int} = \frac{g_2}{2\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1 - \gamma_5)V \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + h.c. \quad (1)$$

in the Lagrange density. Here g_2 is the weak SU(2) coupling, W^μ is the charged W -boson field and V is the Cabibbo–Kobayashi–Maskawa matrix. V arises from the diagonalization of the quark mass matrices. It can be written in terms of three Euler like angles and a complex phase $e^{i\delta}$. In the minimal standard model it is this phase that is responsible for the CP violation observed in kaon decay and CP violation in B decay. Extensions of the standard model with extended Higgs sectors usually have additional sources of CP violation. It is hoped to test the correctness of the minimal standard model for CP violation in future B decay experiments and elsewhere.

In the minimal standard model the elements of the Cabibbo–Kobayashi–Maskawa matrix are fundamental parameters that must be determined from experiment. In this talk I will concentrate on those issues in heavy quark theory that are related to a determination of $|V_{ub}|$ and $|V_{cb}|$ from B decays. Other interesting areas where progress has occurred will, for the most part, be omitted. Even within the area of those elements of heavy quark physics related to determining the weak mixing angles I will not be able to give a complete review. For example, I will not have time to discuss the implications of sum rules in semileptonic decay and lattice QCD results.

In order to present the new developments in the theory of heavy quarks in their proper context and to fully appreciate their significance I will briefly review some of the key early work on heavy quark theory.

2. Heavy Quark Effective Theory

The part of the QCD Lagrange density that contains a heavy quark Q is

$$\mathcal{L} = \bar{Q}(i\not{D} - m_Q)Q . \quad (2)$$

For situations where the heavy quark Q is interacting with light degrees of freedom (i.e., light quarks and gluons) carrying momentum much less than its mass, m_Q , it is appropriate to take the limit $m_Q \rightarrow \infty$ with the heavy quark four-velocity, v^μ , held fixed.¹ In this limit the interactions of the heavy quark become independent of its mass and spin resulting in the approximate heavy quark spin-flavor symmetries of QCD.

To take this limit write

$$(x) = e^{-im_Q v \cdot x} h_v^{(Q)}(x) \quad (3)$$

where

$$\not{v} h_v^{(Q)} = h_v^{(Q)} \quad (4)$$

Putting eq. (3) into (2) gives

$$\mathcal{L}_0 = \bar{h}_v^{(Q)}(i\not{D} + m_Q(\not{v} - 1))h_v^{(Q)} . \quad (5)$$

Using the constraint (4) this can be simplified to^{2,3}

$$\mathcal{L}_0 = \bar{h}_v^{(Q)} i v \cdot D h_v^{(Q)} . \quad (6)$$

Note that the Lagrange density in eq. (6) is independent of the heavy quark's mass and it's spin. Consequently the heavy quark effective theory has a spin flavor symmetry.¹ For charm and bottom quarks moving with the same velocity this is an SU(4) symmetry. Much of the predictive power of the heavy quark effective theory arises because of this symmetry.

The heavy quark field $h_v^{(Q)}$ destroys a quark Q but it does not create the corresponding antiquark. Pair creation does not occur in the heavy quark effective theory.

3. $1/m_Q$ Corrections

The heavy quark effective theory in (6) represents the $m_Q \rightarrow \infty$ limit of QCD. At finite m_Q there are corrections suppressed by powers of $1/m_Q$. These can be included in a systematic fashion. In general

$$Q(x) = e^{-im_Q v \cdot x} [h_v^{(Q)}(x) + \chi_v^{(Q)}(x)] \quad (7)$$

where

$$\not{v} h_v^{(Q)} = h_v^{(Q)} \quad \text{and} \quad \not{v} \chi_v^{(Q)} = -\chi_v^{(Q)} \quad (8)$$

The equation of motion for the heavy quark field Q

$$(i\not{D} - m_Q)Q = 0 \quad (9)$$

can be used to express $\chi_v^{(Q)}(x)$ in terms of $h_v^{(Q)}(x)$ order by order in $1/m_Q$. Putting (7) into (9) and using (8) gives

$$\chi_v^{(Q)} = \frac{1}{2m_Q} i\not{D} [h_v^{(Q)} + \chi_v^{(Q)}] \quad (10)$$

which implies that

$$\chi_v^{(Q)} = \frac{1}{2m_Q} i\not{D} h_v^{(Q)} + \mathcal{O}(1/m_Q^2) . \quad (11)$$

Using this in eq. (7) and then plugging (7) into the Lagrange density (2) gives the heavy quark effective theory including $1/m_Q$ corrections.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 \quad (12)$$

with \mathcal{L}_0 given by eq. (6) and^{4,5}

$$\mathcal{L}_1 = \bar{h}_v^{(Q)} \frac{(iD)^2}{2m_Q} h_v^{(Q)} - a_2(\mu) \bar{h}_v^{(Q)} g \frac{G_{\alpha\beta} \sigma^{\alpha\beta}}{4m_Q} h_v^{(Q)} \quad (13)$$

with $a_2(\mu) = 1$. In eq. (13) g is the strong gauge coupling and $G_{\alpha\beta}$ is the gluon field strength tensor. The procedure we have outlined above amounts to match-

ing tree graphs in QCD with those in the heavy quark effective theory. When loops are included a_2 develops subtraction point dependence because the operator $\bar{h}_v^{(Q)} g G_{\alpha\beta} \sigma^{\alpha\beta} h_v^{(Q)}$ requires renormalization. In the leading logarithmic approximation

$$a_2(\mu) = [\alpha_s(m_Q)/\alpha_s(\mu)]^{9/(33-2n_f)} \quad (14)$$

where n_f is the number of light quark flavors.

The first term in eq. (13) is the heavy quark kinetic energy. It breaks the heavy quark flavor symmetry but not the spin symmetry. The second term in eq. (13) is the energy from the interaction of the heavy quark's color magnetic moment with the chromomagnetic field. It breaks both the spin and flavor symmetries.

4. Spectroscopy of Heavy Hadrons

In the $m_Q \rightarrow \infty$ limit hadrons containing a single heavy quark Q are classified not only by their total spin \vec{S} but also by the spin of their light degrees of freedom⁶

$$\vec{S}_\ell = \vec{S} - \vec{S}_Q . \quad (15)$$

Since $s_Q = 1/2$, in this limit hadrons containing a single heavy quark occur in degenerate doublets labelled by the spin of the light degrees of freedom s_ℓ and with total spins

$$s = s_\ell \pm 1/2 \quad (16)$$

An exception occurs for $s_\ell = 0$ where there is only one state with $s = 1/2$. For mesons with $Q\bar{q}$ ($q = u$ or d) flavor quantum numbers the ground state doublet has negative parity and $s_\ell = 1/2$ giving a doublet of spin-zero and spin-one mesons. For $Q = c$ they are the D and D^* mesons and for $Q = b$ they are the B and B^* mesons. In the $Q = c$ case an excited doublet of positive parity mesons with $s_\ell = 3/2$ has been observed. The hadrons in this doublet are sometimes called D^{**} mesons and have total spins one and two.

Baryons with Qqq flavor quantum numbers have also been observed. The ground state isospin zero baryons have positive parity and $s_\ell = 0$ and are called Λ_Q baryons. The ground state $I = 1$ baryons have positive parity and $s_\ell = 1$ and come in a doublet with $s = 1/2$ and $3/2$. They are called Σ_Q and Σ_Q^* baryons. For $Q = c$ the Λ_c and Σ_c baryons have been observed and for $Q = b$ the Λ_b baryon has been observed. In the charm case two excited baryons have also been observed. Their properties are consistent with being a negative parity doublet of $I = 0$ baryons with $s_\ell = 1$ giving total spins $1/2$ and $3/2$.

The mass of a hadron H_Q containing a single heavy quark Q can be expanded in powers of $1/m_Q$. Up to order $1/m_Q$ it has the form

$$\begin{aligned}
m_{H_Q} = m_Q + \bar{\Lambda} - \langle H_Q | \bar{h}_v^{(Q)} \frac{(iD)^2}{2m_Q} h_v^{(Q)} | H_Q \rangle / 2m_H \\
+ a_2(\mu) \langle H_Q | \bar{h}_\sigma^{(Q)} \frac{gG_{\alpha\beta} \sigma^{\alpha\beta}}{4m_Q} h_v^{(Q)} | H_Q \rangle / 2m_H + \mathcal{O}(1/m_Q^2) .
\end{aligned} \tag{17}$$

The first term on the rhs of equation (17), m_Q , is the heavy quark pole mass. The second $\bar{\Lambda}$ is the mass of the light degrees of freedom in the hadron. It does not depend on the heavy quark mass but does depend on the quantum numbers of the light degrees of freedom. The third term is the heavy quark's kinetic energy and the final term is its chromomagnetic energy. Only the last term depends on the spin of the heavy quark and it causes the splittings in the hadron doublets mentioned earlier. For example

$$m_{B^*} - m_B = -\frac{4}{3} a_2(\mu) \langle B | \bar{h}_v^{(b)} \frac{gG_{\alpha\beta}}{4m_b} \sigma^{\alpha\beta} h_v^{(b)} | B \rangle / 2m_B . \tag{18}$$

The heavy quark pole mass m_Q is not a physical quantity and its perturbative expansion has an infrared renormalon ambiguity of order Λ_{QCD} .^{7,8} Nonetheless, it is very convenient to introduce it. As long as final expressions that are compared with experiment express physical quantities in terms of other physical quantities the fact that the pole mass itself is not really well defined is of no consequence.^{9,10}

5. Exclusive $B \rightarrow D^{(*)} e \bar{\nu}_e$ Decay

The rates for $B \rightarrow D e \bar{\nu}_e$ and $B \rightarrow D^* e \bar{\nu}_e$ are determined by the value of $|V_{bc}|$ and the hadronic matrix element of the weak current $\bar{c} \gamma_\mu (1 - \gamma_5) b$ between B and $D^{(*)}$ states. The application of heavy quark effective theory involves a two step process. First is matching the current $\bar{c} \gamma_\mu (1 - \gamma_5) b$ onto operators in the heavy quark effective theory. In the leading logarithmic approximation this matching takes the simple form¹¹

$$\bar{c} \gamma_\mu (1 - \gamma_5) b = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25} \left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)} \right]^{a_L} \cdot \bar{h}_{v'}^{(c)} \gamma_\mu (1 - \gamma_5) h_v^{(b)} \quad (19)$$

where

$$a_L = \frac{8}{27} [v \cdot v' r (v \cdot v') - 1] \quad (20)$$

and

$$r(v \cdot v') = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ln(v \cdot v' + \sqrt{(v \cdot v')^2 - 1}). \quad (21)$$

Note that for $v \cdot v' \neq 1$ the coefficient of the current in the effective theory $\bar{h}_{v'}^{(c)} \gamma_\mu (1 - \gamma_5) h_v^{(b)}$ depends on the subtraction point μ . In the effective theory where the charm and bottom quarks are both treated as heavy the operator $\bar{h}_{v'}^{(c)} \gamma_\mu (1 - \gamma_5) h_v^{(b)}$ requires renormalization. Its matrix elements have a μ dependence that cancels that of its coefficient. However, at zero recoil $v \cdot v' = 1$ the coefficient is independent of μ . At this kinematic point the operator is the conserved current associated with the spin-flavor symmetries of the heavy quark effective theory and consequently it is not renormalized.

Matrix elements of $\bar{h}_{v'}^{(c)} \Gamma h_v^{(b)}$ in the heavy quark effective theory between B and D^* states are related by heavy quark spin symmetry to a single universal function of $v \cdot v'$,¹

$$\frac{\langle D(v') | \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} | B(v) \rangle}{\sqrt{m_B m_D}} = \xi(v \cdot v') Tr \left\{ \frac{(\not{v}' + 1)}{2} \Gamma \frac{(\not{v} + 1)}{2} \right\} \quad (22)$$

$$\frac{\langle D^*(v', \varepsilon) | \bar{h}_{v'}^{(c)} \Gamma h_v^{(b)} | B(v) \rangle}{\sqrt{m_B m_{D^*}}} = \xi(v \cdot v') \text{Tr} \left\{ \varepsilon^* \frac{(\not{v}' + 1)}{2} \Gamma \frac{(\not{v} + 1)}{2} \gamma_5 \right\}. \quad (23)$$

For $v \cdot v' \neq 1$ the Isgur–Wise function $\xi(v \cdot v')$ depends on the subtraction point μ . However, at zero recoil heavy quark flavor symmetry fixes the normalization^{1,12,13} of ξ ,

$$\xi(1) = 1. \quad (24)$$

Equations (22) and (23) hold in the $m_{c,b} \rightarrow \infty$ limit. In general there are $\Lambda_{QCD}/m_{c,b}$ corrections. However, at zero recoil it has been shown that corrections first arise at order $\Lambda_{QCD}^2/m_{c,b}^2$.^{13,14} This important result opens an avenue for the precise determination of $|V_{cb}|$ from exclusive $B \rightarrow D^* e \bar{\nu}_e$ decay.

Neglecting nonperturbative corrections, suppressed by powers of $(\Lambda_{QCD}/m_{b,c})$, the zero recoil, the matrix elements of the axial, and vector currents are

$$\frac{\langle D(v) | \bar{c} \gamma_\mu b | B(v) \rangle}{\sqrt{m_B m_D}} = 2\eta_V v_\mu \quad (25)$$

$$\frac{\langle D^*(v, \varepsilon) | \bar{c} \gamma_\mu b | B(v) \rangle}{\sqrt{m_B m_{D^*}}} = 2\eta_A \varepsilon_\mu^*, \quad (26)$$

where η_V and η_A are QCD correction factors from matching currents in the full theory onto those in the effective theory. In the leading logarithmic approximation where $\ln(m_b/m_c)$ is treated as large and all terms of order $[\alpha_s \ln(m_b/m_c)]^n$ are summed^{15,16}

$$\eta_V = \eta_A = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right]^{-6/25}. \quad (27)$$

However, since m_b/m_c is not that large a better approximation is to keep the full dependence on m_c/m_b . The coefficients η_V and η_A have been calculated including two loop terms that come from vacuum polarization insertions and are proportional

to

$$\beta^{(0)} = 11 - \frac{2}{3}n_f. \quad (28)$$

The result is^{13,17,18}

$$\eta_V = 1 + \frac{1}{3} \frac{\bar{\alpha}_s(m_b)}{\pi} \phi(m_c/m_b) + \left(\frac{\bar{\alpha}_s(m_b)}{\pi} \right)^2 \left[\frac{1}{72} \phi(m_c/m_b) \beta^{(0)} + \dots \right] + \dots, \quad (29)$$

and

$$\begin{aligned} \eta_A = 1 + \frac{1}{3} \frac{\bar{\alpha}_s(m_b)}{\pi} [\phi(m_c/m_b) - 2] + \left(\frac{\bar{\alpha}_s(m_b)}{\pi} \right)^2 \\ \cdot \left[\left(\frac{5}{72} \phi(m_c/m_b) - \frac{14}{72} \right) \beta^{(0)} + \dots \right] + \dots, \end{aligned} \quad (30)$$

where

$$\phi(z) = -3 \left(\frac{1+z}{1-z} \right) \ln z - 6, \quad (31)$$

m_c and m_b are heavy quark pole masses and $\bar{\alpha}_s$ is the \overline{MS} strong coupling. The ellipses in the square brackets are terms independent of n_f . There are reasons to believe that the order $\bar{\alpha}_s^2(m_b)$ piece proportional to $\beta^{(0)}$ provides a good approximation to the full order $\bar{\alpha}_s^2(m_b)$ term. That is true for $R(e^+e^- \rightarrow \text{hadrons})$, $\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})$ and the relation between the heavy quark pole mass m_Q and the running heavy quark \overline{MS} mass $\bar{m}_Q(m_Q)$:

$$\begin{aligned} R(e^+e^- \rightarrow \text{hadrons}) = 3 \left(\sum_i Q_i^2 \right) \left[1 + \frac{\bar{\alpha}_s(\sqrt{s})}{\pi} \right. \\ \left. + (0.17\beta^{(0)} + 0.08) \left(\frac{\bar{\alpha}_s(\sqrt{s})}{\pi} \right)^2 + \dots \right] \end{aligned} \quad (32)$$

$$\frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{3\Gamma(\tau \rightarrow \nu_\tau \bar{\nu}_e e)} = 1 + \frac{\bar{\alpha}_s(m_\tau)}{\pi} + (0.57\beta^{(0)} + 0.08) \left(\frac{\bar{\alpha}_s(m_\tau)}{\pi} \right)^2 + \dots \quad (33)$$

$$m_Q/\bar{m}_Q(m_Q) = 1 + \frac{4}{3} \frac{\bar{\alpha}_s(m_Q)}{\pi} + (1.56\beta^{(0)} - 1.05) \left(\frac{\bar{\alpha}_s(m_Q)}{\pi} \right)^2 + \dots \quad (34)$$

Evaluating eqs. (29) and (30) with $m_c/m_b = 0.30$ and $\bar{\alpha}_s(m_b) = 0.20$ gives

$$\begin{aligned} \eta_V &= 1 + 0.02 + 0.004 \\ \eta_A &= 1 - 0.03 - 0.005 \quad , \end{aligned} \quad (35)$$

In eqs. (35) the second and third terms are the ones of order $\bar{\alpha}_s$ and $\bar{\alpha}_s^2\beta^{(0)}$ respectively. Also we have taken $n_f = 2$ which gives $\beta^{(0)} = 9$. Note that the two loop term is much smaller than the one loop term indicating that the perturbation series is well behaved.

Nonperturbative corrections to (25) and (26) are of order $(\Lambda_{QCD}/m_{c,b})^{n+2}$, $n = 0, 1, \dots$. For $n = 0$ these have been characterized in terms of matrix elements of various operators in the heavy quark effective theory and estimated using phenomenological models.¹⁹ In addition the corrections to eqs. (25) and (26) that are enhanced by $\ln m_\pi$ or factors of $1/m_\pi$ have been computed using chiral perturbation theory. These have an interesting form.²⁰ The correction of order $(\Lambda_{QCD}/m_{c,b})^2$ is enhanced by $\ln m_\pi$ but corrections suppressed by higher powers $(\Lambda_{QCD}/m_{c,b})^{n+2}$, $n = 1, 2, \dots$ are enhanced by $(\Lambda_{QCD}/m_\pi)^n$. Consequently, power suppressed terms are important for all n . These corrections are calculable in terms of the $D^*D\pi$ coupling. Unfortunately the value of this coupling is not known. This gives one of the major uncertainties in the size of the power correction to eqs. (25) and (26).

6. Inclusive $B \rightarrow X_{c,u} e \bar{\nu}_e$ Decay

Over the past few years there has been great progress in our understanding of inclusive semileptonic B meson decay.^{21,22,23,24} The strong interaction physics relevant for this process is parametrized by the hadronic tensor

$$W_{c,u}^{\mu\nu} = (2\pi)^3 \sum_X \delta^4(p_B - q - p_X) \langle B | J_{c,u}^{\mu\dagger} | X \rangle \langle X | J_{c,u}^\nu | B \rangle \quad (36)$$

and

$$J_c^\mu = \bar{c}\gamma^\mu(1 - \gamma_5)b \quad (37)$$

$$J_u^\mu = \bar{u}\gamma^\mu(1 - \gamma_5)b \quad (38)$$

$W^{\mu\nu}$ can be expanded in terms of scalar form factors $W_n, n = 1, 2, \dots, 5$ that are functions of q^2 and $v \cdot q$.

$$\begin{aligned} W^{\mu\nu} = & -g^{\mu\nu}W_1 + v^\mu v^\nu W_2 - i\varepsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta W_3 \\ & + q^\mu q^\nu W_4 + (q^\mu v^\nu + q^\nu v^\mu)W_5 . \end{aligned} \quad (39)$$

The form factors W_j are the imaginary parts of form factors that occur in the matrix element of the time ordered product of weak currents.

$$T_{c,u}^{\mu\nu} = -i \int d^4x e^{-iq \cdot x} \langle B | T(J_{c,u}^{\mu\dagger}(x) J_{c,u}^\nu(0)) | B \rangle \quad (40)$$

can be expanded in terms of scalar form factors

$$\begin{aligned} T^{\mu\nu} = & -g^{\mu\nu}T_1 + v^\mu v^\nu T_2 - i\varepsilon^{\mu\nu\alpha\beta}v_\alpha q_\beta T_3 \\ & + q^\mu q^\nu T_4 + (q^\mu v^\nu + q^\nu v^\mu)T_5 \end{aligned} \quad (41)$$

and

$$Im T_{c,u} = -\pi W_{c,u}. \quad (42)$$

Predictions for the form factors T_j can be made by performing an operator product expansion and making a transition to the heavy quark effective theory. The leading operator encountered is $\bar{b}\gamma_\mu b$ and its matrix element is known since it is the conserved b-quark number current. Here there is no need to make the transition to the heavy quark effective theory to understand the m_b dependence. There are no dimension four

operators and the dimension 5 operators that occur are the b -quark kinetic energy and the chromomagnetic dipole term that occur in \mathcal{L}_1 of eq. (13). Consequently at leading order in $1/m_b$ the differential decay rate $d\Gamma/dq^2 dE_e$ for inclusive semileptonic B -decay is given by free b -quark decay. There are no nonperturbative corrections of order (Λ_{QCD}/m_b) . Nonperturbative corrections of order $(\Lambda_{QCD}/m_b)^2$ are characterized by the two dimensionless parameters

$$K_b = -\langle B | \bar{h}_v^{(b)} \frac{(iD)^2}{2m_b^2} h_v^{(b)} | B \rangle / 2m_B \quad (43)$$

$$G_b = a(\mu) \langle B | \bar{h}_v^{(b)} \frac{g G_{\mu\nu}}{4m_b^2} \sigma^{\mu\nu} h_v^{(b)} | B \rangle / 2m_B . \quad (44)$$

Including perturbative corrections and nonperturbative corrections suppressed by $(\Lambda_{QCD}/m_b)^2$ the $B \rightarrow X_c e \bar{\nu}_e$ semileptonic decay rate is

$$\begin{aligned} \Gamma(B \rightarrow X_c e \bar{\nu}_e) = & \Gamma_0 [(1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln \rho) \eta_{incl} \\ & + K_b (-1 + 8\rho - 8\rho^3 + \rho^4 + 12\rho^2 \ln \rho) + G_b (3 - 8\rho + 24\rho^2 - 24\rho^3 \\ & + 5\rho^4 + 12\rho^2 \ln \rho)] \end{aligned} \quad (45)$$

where

$$\rho = m_c^2/m_b^2, \quad (46)$$

and

$$\Gamma_0 = \frac{|V_{cb}|^2 G_F^2 m_b^5}{192\pi^3} . \quad (47)$$

In Γ_0 m_b is the b -quark pole mass and η_{incl} gives the effects of perturbative QCD corrections. Results for $B \rightarrow X_u e \bar{\nu}_e$ are obtained by taking $\rho = 0$ and $V_{cb} \rightarrow V_{ub}$. η_{incl} depends on m_c/m_b so the perturbative QCD corrections are different for $B \rightarrow X_c e \bar{\nu}_e$ and $B \rightarrow X_u e \bar{\nu}_e$ decay. The nonperturbative corrections are quite small.

Furthermore, G_b is known from the $B^* - B$ mass splitting so the only uncertainty in the nonperturbative corrections comes from the size of K_b . In eqs. (45)-(47) m_c and m_b are the charm and bottom quark pole masses. If m_c is eliminated by

$$m_B - m_D = m_b - m_c + m_b K_b - m_c K_c + m_b G_b - m_c G_c, \quad (48)$$

then the decay rate is not too sensitive to the value of m_b . For example, as m_b varies between 5GeV and 4.5GeV the rate $\Gamma(B \rightarrow X_c e \bar{\nu}_e)$ changes by only 20%.

Neglecting the nonperturbative corrections the B decay rate equals the b -quark decay rate. The perturbative QCD corrections of order $\bar{\alpha}_s(m_b)$ have been computed and those of order $\bar{\alpha}_s(m_b)^2$ proportional to $\beta^{(0)}$ are also known. We write

$$\begin{aligned} \eta_{incl} = 1 - \left(\frac{\bar{\alpha}_s(m_b)}{\pi} \right) \frac{2}{3} \left(\pi^2 - \frac{25}{4} + \delta_1(m_c/m_b) \right) \\ - \left(\frac{\bar{\alpha}_s(m_b)}{\pi} \right)^2 (\beta^{(0)} \chi_\beta(m_c/m_b) + \dots) + \dots \quad . \end{aligned} \quad (49)$$

The function $\delta_1(x)$ is known analytically.²⁵ It takes into account the effect of the charm quark mass on the order $\bar{\alpha}_s$ QCD corrections; $\delta_1(0) = 0$. Numerically $\delta_1(0.3) = -1.11$. The function $\chi_\beta(x)$ has been determined numerically yielding $\chi_\beta(0) = 3.2$ and $\chi_\beta(0.3) = 1.7$. Using $\bar{\alpha}_s(m_b) = 0.20$ and $m_c/m_b = 0.30$ gives²⁶

$$\eta_{incl} = 1 - 0.11 - 0.06 + \dots \quad (50)$$

for $B \rightarrow X_c e \bar{\nu}_e$ decay and (using $m_c/m_b = 0$)

$$\eta_{incl} = 1 - 0.15 - 0.11 + \dots \quad , \quad (51)$$

for $B \rightarrow X_u e \bar{\nu}_e$ decay. The second and third terms in eqs. (50) and (51) are the pieces of order $\bar{\alpha}_s(m_b)$ and $\bar{\alpha}_s(m_b)^2 \beta^{(0)}$ respectively. In the “two loop” term we have taken $n_f = 2$ which gives $\beta^{(0)} = 9$. For $B \rightarrow X_u e \bar{\nu}_e$ the perturbative series

is not well behaved and the situation for $B \rightarrow X_c e \bar{\nu}_e$ is somewhat marginal. For inclusive semileptonic $D \rightarrow X_s \bar{e} \nu_e$ decay similar formulas hold. The perturbative QCD corrections can be deduced from eq. (49) with $\bar{\alpha}_s(m_b) \rightarrow \bar{\alpha}_s(m_c)$ and $m_c/m_b \rightarrow m_s/m_c \simeq 0$. Here the QCD corrections are also not under control.

The methods outlined above for inclusive semileptonic B decay can also be applied to nonleptonic B -decay. Here one runs into a potential conflict between the measured semileptonic branching ratio and the measured charm multiplicity.^{27,28} For the decays that come from $b \rightarrow c \bar{c} s$ the charm quark masses take up most of the available energy. Therefore, it is not clear that local duality can be used to relate the quark level decay to the hadron decay. Furthermore, the perturbative QCD corrections in the quark level decay may not be under control. To accommodate the measured semileptonic branching ratio²⁹ $B_{SL} = (10.4 \pm 0.4)\%$ requires about 40% of the nonleptonic B decays to come from the $b \rightarrow c \bar{c} s$ mechanism. This implies a charm multiplicity $\langle n_c \rangle \simeq 1.3$. However, the measured charm multiplicity³⁰ is only $\langle n_c \rangle_{\text{exp}} = 1.04 \pm 0.07$. It will take more data to resolve this issue.

7. The End Point Region of the Electron Spectrum

The maximum electron energy in the exclusive decay $B \rightarrow X e \bar{\nu}_e$ is

$$E_e^{\text{max}} = \frac{m_B^2 - m_X^2}{2m_B} . \quad (52)$$

Therefore, semileptonic B decays with electron energies greater than $(m_B^2 - m_D^2)/2m_B$ must have come from a $b \rightarrow u$ transition. This endpoint region of the electron energy spectrum is very important. Understanding it in a model independent way may lead to a precise determination of V_{ub} .

For inclusive $B \rightarrow X_u e \bar{\nu}_e$ decay the electron energy spectrum, including nonperturbative effects of order $(\Lambda_{QCD}/m_b)^2$, has been found using the operator product expansion methods outlined in the previous section. Neglecting perturbative QCD

corrections^{22,23}

$$\begin{aligned} \frac{1}{\Gamma_0} \frac{d\Gamma}{dy} = & \left[2(3-2y)y^2 - \frac{20}{3}y^3K_b - \left(8 + \frac{20}{3}y\right)y^2G_b \right] \theta(1-y) \\ & + \frac{2}{3}[K_b + 11G_b]\delta(1-y) + \frac{2}{3}K_b\delta'(1-y) , \end{aligned} \quad (53)$$

where

$$y = (2E_e/m_b) , \quad (54)$$

and K_b and G_b are given in eqs. (43) and (44). These matrix elements are of order ε^2 where,

$$\varepsilon = \Lambda_{QCD}/m_b . \quad (55)$$

The maximum electron energy for b -quark decay is $y = 1$ (i.e., $E_e = m_b/2$). However, nonperturbative effects (e.g., motion of the b -quark in the B -meson) extend the maximum electron energy for B -meson decay beyond this point. Since we are treating such effects as a power series in ε they are represented by singular terms at $y = 1$. To all orders in ε the decay spectrum obtained from the operator product expansion has the structure³¹ (at zero'th order in $\alpha_s(m_b)$)

$$\begin{aligned} \frac{1}{\Gamma_0} \frac{d\Gamma}{dy} = & \theta(1-y)(\varepsilon^0 + 0\varepsilon + \varepsilon^2 + \dots) \\ & + \delta(1-y)(0\varepsilon + \varepsilon^2 + \dots) + \delta^{(1)}(1-y)(\varepsilon^2 + \varepsilon^3 + \dots) + \dots \\ & + \delta^{(n)}(1-y)(\varepsilon^{n+1} + \varepsilon^{n+2} + \dots) + \dots , \end{aligned} \quad (56)$$

where ε^n denotes a term of that order, which may include a smooth function of y . In eq. (56) $\delta^{(n)}(1-y)$ denotes the n 'th derivative of $\delta(1-y)$ with respect to y . The contribution to the total decay rate of a term in $d\Gamma/dy$ of order $\varepsilon^n \delta^{(n)}(1-y)$ is of order ε^n .

The semileptonic decay width for $b \rightarrow u$ is difficult to measure because of background contamination from the dominant $b \rightarrow c$ semileptonic decays. It is therefore, important to be able to compute the rate in the endpoint region near $y = 1$. One way to calculate the endpoint spectrum is to weight the differential decay distribution $d\Gamma/dy$ in eq. (56) by a normalized function of width σ around $y = 1$. We refer to this process as smearing. Most of the details of the smearing procedure are unimportant; the only quantity of relevance is the width σ of the smearing region.

The singular distribution $\varepsilon^m \delta^{(n)}(1 - y)$ (where $m > n$) smeared over a region of width σ gives a contribution of order $\varepsilon^n / \sigma^{n+1}$ to $d\Gamma/dy$. If the width σ of the smearing region is of order ε^p the generic term $\varepsilon^m \delta^{(n)}(1 - y)$ yields a contribution of order $\varepsilon^{m-(n+1)p}$. Since $m > n$ this shows that the $1/m_b$ expansion for the spectrum breaks down unless $p \leq 1$, i.e., the smearing region cannot be narrower than ε . The divergence for $p > 1$ is not associated with the failure of the operator product expansion due to resonances with masses of order the QCD scale. The region of the electron energy spectrum for which such resonances dominate the final hadronic states is of width ε^2 , while the expansion breaks down upon smearing over any region of size $\varepsilon^{1+\delta}$, where $\delta > 0$.

If the smearing region is chosen of order ε the form of the expansion in eq. (56) shows that the leading terms of the form $\theta(1 - y)$ and $\varepsilon^{n+1} \delta^{(n)}(1 - y)$ all contribute at order unity to the smeared spectrum. Thus one can obtain the decay spectrum smeared over a width ε if the leading singularities can be summed. The sum of the leading singularities produces a distribution $d\Gamma/dy$ of width ε and height of order unity (i.e., of the same order as the free quark distribution). Neubert and Bigi, et al., have shown how to sum the leading singularities.^{32,33} They are characterized by the matrix elements

$$\frac{1}{2m_B} \langle B | \bar{h}_v^{(b)} iD^{\mu_1} \dots iD^{\mu_n} h_v^{(b)} | B \rangle = A_n v_{\mu_1} \dots v_{\mu_n} + \dots \quad (57)$$

The ellipsis on the right side of eq. (57) denote other Lorentz structures. For example,

with $n = 2$ the matrix element is,

$$\frac{1}{2m_B} \langle B | \bar{h}_v^{(b)} iD^{\mu_1} iD^{\mu_2} h_v^{(b)} | B \rangle = A_2 (v_{\mu_1} v_{\mu_2} - g_{\mu_1 \mu_2}) \quad , \quad (58)$$

since $v \cdot Dh_v^{(b)} = 0$. Contracting on μ_1 and μ_2 gives

$$A_2 = \frac{2}{3} m_b K_b \quad . \quad (59)$$

Heavy quark symmetry implies that $A_0 = 1$ and the equation of motion $v \cdot Dh_v^{(b)} = 0$ implies that $A_1 = 0$. The quantities A_n have dimensions of mass to the power n . In terms of them the sum of the leading singularities in the electron spectrum is characterized by a shape function $S(y)$

$$\frac{1}{\Gamma_0} \frac{d\Gamma}{dy} = 2y^2(3 - 2y)S(y) \quad (60)$$

$$S(y) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{A_n}{m_b^n} \right) \frac{d^n}{dy^n} \theta(1 - y) \quad . \quad (61)$$

Perturbative QCD corrections are also singular in the endpoint region. Summing the leading perturbative QCD singularities (i.e., the Sudakov double logarithms) changes the shape function to³¹

$$S(y) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left(\frac{A_n}{m_b^n} \right) \frac{d^n}{dy^n} R(y) \quad (62)$$

where

$$R(y) = \exp \left[-\frac{2}{3\pi} \alpha_s \ln^2(1 - y) \right] \quad . \quad (63)$$

Recently Korchemsky and Sterman have shown how to sum all the singular perturbative QCD corrections.³⁴

Unfortunately the quantities A_n are not known. However, the same quantities characterize the endpoint photon spectrum in $B \rightarrow X_s \gamma$. So there is hope that a detailed study of the photon spectrum in $B \rightarrow X_s \gamma$ will determine the endpoint region of the electron spectrum in B decays.^{33,34,35}

The methods outlined in this section for describing the endpoint region of the electron spectrum apply when this region is dominated by many states with masses of order $\sqrt{m_b \Lambda_{QCD}}$. In the ISGW³⁶ model the endpoint region where $b \rightarrow c$ transitions are forbidden is dominated by the single decay mode $B \rightarrow \rho e \bar{\nu}_e$. If ρ dominance is found to hold experimentally then the sum of the leading singularities is not a valid description of a region of electron energy which is as small as the difference between the $B \rightarrow X_u e \bar{\nu}_e$ and $B \rightarrow X_c e \bar{\nu}_e$ end points.

If the endpoint region is dominated by the rho meson there are other avenues available to determine V_{ub} . For example, exclusive B and D decays can be used. For $D \rightarrow \rho e \bar{\nu}_e$ the weak mixing angles are known and the form factors for this decay mode to determine them for $B \rightarrow \rho e \bar{\nu}_e$. Using heavy quark symmetry and isospin symmetry³⁷

$$\langle \rho(k) | \bar{u} \gamma_\mu (1 - \gamma_5) b | B \rangle / \sqrt{2m_B} = \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)} \right)^{-6/25} \langle \rho(k) | \bar{d} \gamma_\mu (1 - \gamma_5) c | D \rangle / \sqrt{2m_D} . \quad (64)$$

In the above perturbative QCD effects have been included in the leading logarithmic approximation. If light quark SU(3) symmetry is applied instead of isospin symmetry then the decay $D \rightarrow K^* e \bar{\nu}_e$ can be used (instead of the Cabibbo suppressed decay $D \rightarrow \rho e \bar{\nu}_e$). The form factors for this decay have already been measured. Some problems with this approach are the presence of $1/m_{c,b}$ corrections and possibly large higher order perturbative QCD corrections.³⁸

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